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Limits of the D-brane action

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Abstract: For background geometries whose metric contain a scale γ we reformulate the Born-Infeld D-brane action in terms of $\varepsilon \equiv \gamma/(2\pi\alpha')$. This may be taken as a starting point for various perturbative treatments of the theory. We study two limits that arise at zeroth order of such perturbations. In the first limit, that corresponds to the $g_s \to \infty$ with ε fix, we find a "string parton" picture, also in the presense of some background RR-fields. In the second limit, $\varepsilon \to 0$, we find a topological model.

Keywords: D-branes, String Duality.

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1. Introduction

In this brief letter we study two limits of the Born-Infeld D-brane action. We base the treatment on an expansion in a rescaled tension ε, and include a full background.

We first take a new look at a "strong coupling" limit ($g_s \to \infty$). This is intended as a supplement to the discussion in [1] and [2] where the D-brane world volume was shown to be foliated by string world sheets in this limit. The result is a corroboration of the foliation picture plus the modified equations involving the background n-forms. In particular, we give an example showing that the allowed "string partons" are either infinite (which might correspond to D1-branes), closed or open strings ending on other D-branes (sources of the RR-fields). We also show that the string theory must be of type-IIB.

The second limit we present is $\varepsilon \to 0$. Here we give an ε expansion to first order and discover that the leading term is topological, i.e. independent of the metric. For the case when the dilaton and the 0-form are constant it is proportional to the Chern class of the U(1)-field.

We suggest that both these limits might serve as suitable starting points for investigating D-branes order by order in a parameter expansion. To do so one needs to specify the background, however, and we leave this as a topic for future studies.

2. Background

The long wavelength limit of D-brane dynamics is described by the Born-Infeld action

$$S_{BI}(T_p) = T_p \int d^{p+1}\xi e^{-\Phi'} \sqrt{-\det(\gamma_{ij} + \mathcal{F}_{ij})}\,, \tag{2.1}$$

where $\gamma_{ij} \equiv \partial_i X^\mu \partial_j X^\nu G_{\mu\nu}(X)$ is the metric on the world-sheet induced from a background metric $G_{\mu\nu}$ and $\mathcal{F}_{ij}$ is a world-volume two-form,

$$\mathcal{F}_{ij} \equiv 2\pi\alpha' \partial_{[i}A_{j]} + B_{ij}\,. \tag{2.2}$$

Here $B_{ij} \equiv \partial_i X^\mu \partial_j X^\nu B_{\mu\nu}$ is the pull-back of the (dimensionless) background Kalb-Ramond field. The p-brane tension T_p is related to the fundamental string tension $T \equiv (2\pi\alpha')^{-1}$ and the string coupling constant $g_s \equiv e^{\Phi_\infty}$ by

$$T_p \equiv \frac{1}{(2\pi)^p g_s (\sqrt{\alpha'})^{p+1}}\,, \tag{2.3}$$

with Φ_∞ being the dilaton expectation value and $\Phi' \equiv \Phi - \Phi_\infty$.

We shall be interested in the case when the background metric $G_{\mu\nu}$ contains a parameter γ of dimension (length)^2. As described in [3] this can then be used to introduce a perturbative scheme in the rescaled string tension[1] $\varepsilon \equiv \gamma/2\pi\alpha'$. The parameter γ may, e.g., be (the square of) the radius of (anti)-de Sitter space. In general γ will be choosen from the parameters in the string moduli space, and this choice will not be unique.

Rescaling the fields according to

$$\tilde{\xi}^i = \gamma^{-1/2}\xi^i\,, \qquad \tilde{X}^\mu = \gamma^{-1/2}X^\mu\,, \qquad \tilde{A}_\mu = \gamma^{1/2}A_\mu\,, \tag{2.4}$$

and including the additional coupling to the various (dimensionless) antisymmetric forms $C_{\mu_1\ldots\mu_k}$, the D-brane action in terms of dimensionless fields reads

$$S = \tilde{T}_p \left[\int d^{p+1}\tilde{\xi} e^{-\Phi'} \sqrt{-\det(\tilde{\gamma}_{ij} + \tilde{\mathcal{F}}_{ij})} + g_s \int \tilde{C} \wedge e^{\tilde{\mathcal{F}}} \right], \tag{2.5}$$

where

$$\tilde{T}_p \equiv \frac{\varepsilon^{\frac{p+1}{2}}}{g_s (2\pi)^{\frac{p-1}{2}}}\,, \qquad \tilde{\mathcal{F}}_{ij} = \frac{1}{\varepsilon}\tilde{\partial}_{[i}\tilde{A}_{j]} + \tilde{B}_{ij}\,. \tag{2.6}$$

The action (2.5) is written in the string frame. The dilaton (coupling constant) dependence of the brane-action is frame dependent. We choose to consider the string frame since here the mass of fundamental strings is $\mathcal{O}(1)$ in the string coupling and thus unaffected by our strong coupling limit. Other limits have been considered, however [4].

[1] Calling ε the tension is really a misnomer. It is a dimensionless entity and only related to the fundamental string tension via a rescaling. We hope that this will not cause any confusion.

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3. A strong coupling limit

In this section we will study a limit of the D-brane action when $g_s \to \infty$ in the context of type-IIB string theory.

In general, to investigate the strong coupling limit of string- or M theory one would like to consider the limit of the action (2.5) for large g_s in conjunction with a transformation of the background [5]. We will address the simpler problem of taking the limit $g_s \to \infty$, or more precisely $\varepsilon^{\frac{p+1}{2}}/g_s \ll 1$, $\varepsilon = \text{const.}$, in a fixed background.

The restriction to type-IIB theory is now evident from the following: in type-IIA string theory, because of its $11D$ M-theory origin, the string coupling g_s is related to the radius R_{11} of the compactified dimension through $g_s^{2/3}\sqrt{\alpha'} = R_{11}$. Hence $g_s^{4/3} = \varepsilon$, (identifying $\gamma = R_{11}^2/2\pi$ in the definition of ε above), and it is inconsistent to let $g_s \to \infty$ keeping ε fixed.

Using the method employed in [1] and [2], we find the desired limit to be (dropping tildes),

$$S = \frac{1}{2}\int d^{p+1}\xi \left[V^i W^j(\gamma_{ij} + \mathcal{F}_{ij})\right] + \varepsilon^{\frac{p+1}{2}} \int C \wedge e^{\mathcal{F}} \,, \tag{3.1}$$

where $V^i(\xi)$ and $W^j(\xi)$ are world volume vector densities. This may be considered as the zeroth order term in an expansion of the action (2.5) in powers of g_s^{-1}. We will study it in its own right, though.

The V^i and W^j-field equations derived from (3.1) read

$$\begin{aligned} W^j(\gamma_{ij} + \mathcal{F}_{ij}) &= 0\,, \\ V^i(\gamma_{ij} + \mathcal{F}_{ij}) &= 0\,, \end{aligned} \tag{3.2}$$

and the integrability condition[2] for these equations is

$$\det(\gamma_{ij} + \mathcal{F}_{ij}) \equiv \det(\gamma_{ij} + B_{ij} + \varepsilon^{-1} F_{ij}) = 0\,. \tag{3.3}$$

To gain a first qualitative understanding of this relation, we set $B = 0$. In this case it is equivalent to

$$\det(\delta^i_j + \varepsilon^{-1} F^i{}_j) = 0\,. \tag{3.4}$$

In analogy to the usual treatment of the Born-Infeld action one might be tempted to employ a weak gradient expansion [6] and expand (3.4) in a power series in $|\partial A| \ll 1$. However, as is easily seen, this implies that $\varepsilon^2 \propto \operatorname{tr} F^2 \ll 1$, which is not true in general. E.g., when the background is Minkowski space we have $\varepsilon \propto \infty$.[3] We thus learn that the weak gradient expansion is not in general viable.

[2]We shall not be interested in the trivial solution $V^i = W^i = 0$. This corresponds to a theory with an WZ action given by the second term on the r.h.s. of (3.1).

[3]This results if we think of Minkowski space as the $R \to \infty$ limit of de Sitter space. In an alternative scheme discussed in [3] ε is of order 1 which also violates our inequality.

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3.1 Solving the VW-equations

Although the equation (3.3) says that the combined matrix $\gamma + \mathcal{F}$ is degenerate, this is not so for the induced metric γ itself. We may thus introduce a vielbein e_a^i and its inverse e_i^a on the world-volume according to

$$e_i^a e_j^b \eta_{ab} = \gamma_{ij} \,, \qquad a, b = 0, \ldots, p \,, \tag{3.5}$$

where η_{ab} is the $p+1$-dimensional Minkowski metric. In [1] it was found that the first part of the action (3.1) has a two-dimensional Lorentz structure. This may be displayed by a redefinition of V^i and W^i. With e^{-1} being the determinant of the inverse vielbein, we use the SO$(p,1)$-tangent space group to write[4]

$$\begin{aligned} V^i &= (e_0^i + e_1^i)\sqrt{e^{-1}} \equiv (\tilde{e}_0^i + \tilde{e}_1^i) \,, \\ W^i &= (e_0^i - e_1^i)\sqrt{e^{-1}} \equiv (\tilde{e}_0^i - \tilde{e}_1^i) \,, \end{aligned} \tag{3.6}$$

thus breaking the tangent-space group down to SO$(1,1) \times$ SO$(p-2)$. In this gauge, (3.2) reads

$$\mathcal{F}_{ij} e_0^j = \gamma_{ij} e_1^j \,, \qquad \mathcal{F}_{ij} e_1^j = \gamma_{ij} e_0^j \,, \tag{3.7}$$

or, equivalently,

$$\left(\mathcal{F}^2\right)^i{}_j \, e_{0,1}^j = e_{0,1}^i \,, \tag{3.8}$$

with the integrability condition

$$\det\left(\delta_j^i - \left(\mathcal{F}^2\right)^i{}_j\right) = 0 \,. \tag{3.9}$$

Expanding the two-form $\mathcal{F}$ in the bivector basis

$$\mathcal{F} = \frac{1}{2}\mathcal{F}_{ij} d\xi^i \wedge d\xi^j = \frac{1}{2}\mathcal{F}_{ab} e^a \wedge e^b \,, \tag{3.10}$$

(3.7) is solved by

$$\mathcal{F} = \mathcal{F}^{\|} + \mathcal{F}^{\perp} = 2e^0 \wedge e^1 + \mathcal{F}_{AB} e^A \wedge e^B \,, \qquad A, B = 2, \ldots, p \,, \tag{3.11}$$

or, equivalently,

$$\begin{aligned} \nabla_{[0} A_{1]} &= \varepsilon(1 - B_{01}) \,, & \nabla_{[0} A_{A]} &= -\varepsilon B_{0A} \,, \\ \nabla_{[1} A_{A]} &= -\varepsilon B_{1A} \,, & \mathcal{F}_{AB} &= \varepsilon^{-1} \nabla_{[A} A_{B]} + B_{AB} \,, \end{aligned} \tag{3.12}$$

where $\nabla \equiv \partial + \omega \cdot M$ is the covariant derivative, ω being the spin-connection and M the Lorentz-group generator.

The integrability condition (3.9) is of course identically satisfied by (3.11).

We thus learn from the V and W equations that the 2-form $\mathcal{F}$ splits into one part which lies in the $2D$ tangent space spanned by e^0 and e^1, and one part which lies entirely in the orthogonal part spanned by the e^A's. As seen from (3.12), when $B = 0$, this implies that the D-brane electric field is constant and lies in the e_1 direction.

[4] We introduce the tilde notation for later convenience.

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3.2 The A-equations

The A field equations that follow from (3.1) are

$$\partial_i \left(V^{[i} W^{j]} \right) = \mathcal{J}^j \,, \tag{3.13}$$

where $\mathcal{J}^i$ represents the contribution from the background RR gauge fields and is given by

$$\mathcal{J}^i \equiv -\varepsilon^{(p+1)/2} \sum_{s=1}^{(p+1)/2} \frac{1}{(s-1)!2^{s-1}} {}^*H^{il_1k_1\cdots l_{s-1}k_{s-1}} \mathcal{F}_{l_1k_1} \cdots \mathcal{F}_{l_{s-1}k_{s-1}} \,, \tag{3.14}$$

where ${}^*H^{l_1\cdots l_{p-n}}$ is the dual of the pull-back of the field strength of the $C^{(n)}$ form.

In [1] and [2] the structure of the theory was shown to be simplified in the following diffeomorphism gauge:

$$\partial_i V^i = \partial_i W^i = 0 \,. \tag{3.15}$$

In that gauge (3.13) may be rewritten as

$$[V, W]^i = \mathcal{J}^i \,, \tag{3.16}$$

where $[\cdot,\cdot]$ denotes the Lie-bracket.

The equations (3.13) are highly non-linear. To proceed further we need to make some simplifying assumptions, and we first consider the case when the background RR gauge fields are zero ($C^{(n)} = 0$). In that case (3.15) says that $[V^i\partial_i, W^j\partial_j] = 0$ which means that those directions may be choosen as coordinate-directions [1]. In view of (3.6) we choose to use the $\tilde{e}$ directions instead. Explicitly, we thus use a diffeomorphism gauge where[5]

$$e_a^i = \left\{ e^{1/2}\delta_0^i, e^{1/2}\delta_1^i, e_A^i \right\} , \tag{3.17}$$

which has the inverse

$$\begin{gathered} e_i{}^a = \left\{ e^{-1/2}\delta_0^a, e^{-1/2}\delta_1^a, e_M^a \right\} , \qquad M = 2, \ldots, p \\ e_M^A e_A{}^N = \delta_M^N \,, \qquad e_A{}^M e_M^B = \delta_A^B \,, \qquad e_M{}^{0,1} = -e^{-1/2} e_M^A e_A{}^{0,1} \,. \end{gathered} \tag{3.18}$$

Again setting $B = 0$ and returning to equation (3.7), we find

$$F^i{}_0 = \varepsilon\delta_1^i \,, \qquad F^i{}_1 = \varepsilon\delta_0^i \,. \tag{3.19}$$

We see that (the mixed components of) the U(1) curvature now splits into one part in the $2D$-plane spanned by ξ^0 and ξ^1, and one part in the complementary directions spanned by the ξ^M's. Alternatively, the covariant components are, (setting $e = 1$,)

$$F_{i_0} = \varepsilon\partial_i X^\mu \partial_1 X^\nu G_{\mu\nu} \,, \qquad F_{i1} = \varepsilon\partial_i X^\mu \partial_0 X^\nu G_{\mu\nu} \,, \tag{3.20}$$

[5] We may also use the residual symmetry preserving (3.15) to set $e = 1$.

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with F_{AB} arbitrary. This is the $\mathcal{F}$-solution. This gauge also leads the following expressions that result from (3.5):

$$\begin{aligned} \dot{X}^\mu \dot{X}^\nu G_{\mu\nu} + X'^\mu X'^\nu G_{\mu\nu} &\equiv \gamma_{00} + \gamma_{11} = \eta_{00} + \eta_{11} = 0\,, \\ \dot{X}^\mu X'^\nu G_{\mu\nu} &\equiv \gamma_{01} = \eta_{01} = 0\,, \end{aligned} \tag{3.21}$$

i.e. the Virasoro constraints, (parametrized by $\xi^i, i \neq 0, 1$), in conformal gauge.

3.3 The X^μ-equations

The X^μ field equations that follow from (3.1) are

$$\partial_i \left((V^{(i} W^{j)} G_{\mu\nu} + V^{[i} W^{j]} B_{\mu\nu}) \partial_j X^\nu \right) - V^i W^j \partial_i X^\rho \partial_j X^\nu (G_{\rho\nu} + B_{\rho\nu})_{,\mu} = I_\mu + K_\mu\,, \tag{3.22}$$

where I_μ contains the contribution from the RR gauge fields

$$\begin{aligned} I_\mu \equiv \varepsilon^{(p+1)/2} \sum_{s=0}^{(p+1)/2} & \frac{1}{(p+1-2s)!2^s s!} \partial_{i_1} X^{\mu_1} \partial_{i_2} X^{\mu_2} \cdots \partial_{i_{(p+1-2s)}} X^{\mu_{(p+1-2s)}} \times \\ & \times \epsilon^{i_1 \cdots i_{(p+1-2s)} j_1 k_1 \cdots j_s k_s} H_{\mu\mu_1 \cdots \mu_{(p+1-s)}} \mathcal{F}_{j_1 k_1} \cdots \mathcal{F}_{j_s k_s}\,, \end{aligned} \tag{3.23}$$

and K_μ contains the B-field strength. Setting the C^n's to zero and using the gauges (3.6) and (3.17), the equation (3.22) becomes [2]

$$\partial_0^2 X^\mu - \partial_1^2 X^\mu + \Gamma_{\nu\rho}{}^\mu \partial_0 X^\nu \partial_0 X^\rho - \Gamma_{\nu\rho}{}^\mu \partial_1 X^\nu \partial_1 X^\rho = 0\,, \tag{3.24}$$

i.e. the equation of motion of a string in the 01-coordinates in conformal gauge. Since we also have the Virasoro constraints (3.21), we see that the stringy picture is complete.

3.4 Comments

The interpretation of the strong coupling limit that arises from the above considerations when $C^{(n)} = 0$ is as follows: the world-volume is foliated by world-sheets of (generally (p, q) charged) strings. These strings are either closed or infinite. The form $\mathcal{F}$ is likewise split into one component in the world-sheet direction and the rest in the complementary directions. When $B = 0$ this splitting applies to the U(1) field-strength itself: the component in the world-sheet is then determined by the string tension ε and those in the complementary directions are not determined by the field equations, in agreement with the role of the A-field as a Lagrange multiplier (in this limit).

To gain some further insight into the "string parton" picture one may argue as follows: if instead of the gauge choice (3.17) we only make a partial choice $\tilde{e}_0^i = \delta_0^i$, we find from the A-equations (3.13) that (in the absence of RR-fields),

$$\partial_0 \tilde{e}_1^m = \partial_m \tilde{e}_1^m = 0\,, \qquad m \equiv 1, \ldots, p\,. \tag{3.25}$$

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Thus $\tilde{e}_1^m$ is independent of ξ^0 and divergence-free in the spatial indices m. This means that $\tilde{e}_1^m$ has no sources and that its field lines are either closed or go off to infinity. Since the relations (3.15) still hold in this gauge, we retain the string interpretation with the $\tilde{e}_1^m$-direction representing the spatial string direction. We thus conclude that the strings are either closed or infinite. The strings going off to infinity might possibly have an interpretation as D1-branes (corresponding to branes within branes).

When $C^{(n)} \neq 0$, the situation will in general be more complicated. The one exception is when only $C^{(p+1)} \neq 0$, since this field does not enter in (3.16). The above analysis thus goes through, but now there is a $C^{(p+1)}$-dependent source on the right-hand side of (3.22). When $C^{(n)} \neq 0$, $n < p+1$, the equations become very hard to analyze, in general. There are special cases, however, where an analysis similiar to the one above is possible. We now discuss one such example.

Staying in the gauge $\tilde{e}_0^i = \delta_0^i$ and making the further gauge choice $\partial_0 \tilde{e}_1^0 = 0$, we find from the A-equations (3.13) that

$$[\tilde{e}_0, \tilde{e}_1]^0 = 0\,, \qquad [\tilde{e}_0, \tilde{e}_1]^m = \mathcal{J}^m\,. \tag{3.26}$$

Hence only the spatial components of the currernt (3.14) , $\mathcal{J}^m$, enter the commutation relation, not $\mathcal{J}^0$. Clearly, in backgrounds where $\mathcal{J}^m = 0$ we may again use the coordinates (3.17) (with $e = 1$ for simplicity), and the string interpretation is again viable.

To gain an understanding of the circumstances that might yield such a result, let us consider the current (3.14) for the special case when only $C^{(p+1)}$ and $C^{(p-1)} \neq 0$. The first of these will not affect (3.26), as discussed above. The statement that $\mathcal{J}^m = 0$ is then tantamount to ${}^*H^m = 0$. A sufficient condition for this to be true is that

$$\partial_0 X^{\mu_1} H_{\mu_1 \cdots \mu_p} = 0\,. \tag{3.27}$$

This last condition has a rather clear physical meaning: since we may now go to coordintes (3.17) where the string interpretation holds, we can restate it as $P^{\mu_1} H_{\mu_1 \cdots \mu_p} = 0$, i.e. the fieldstrength $H^{(p)}$ is orthogonal to the momentum-density P of the "parton" string. Since furthermore our gauge choice $\partial_0 e_1^0 = 0$ implies that $\partial_m e_1^m = \mathcal{J}^0$, the argument below (3.25) extends to say that in this case the "string partons" are either infinite strings, closed strings or open strings that end on the D-branes that are sources for $C^{(p+1)}$ and $C^{(p-1)}$. The last case requires that these sources intersect with the world-volume of the D-brane under discussion [7].

4. The $\varepsilon \to 0$ limit

In this section we turn to the $\varepsilon \to 0$ limit of the action (2.5). This may be interpreted as either a small tension or a large curvature limit. The latter interpretation is of particular interest for the AdS/CFT correspondence [8], where the limit away from

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weak gravitation corresponds to a limit away from large N in the boundary theory. In any case $\varepsilon \to 0$ represents a high-energy limit where higher derivative terms, i.e. derivatives of the U(1) field strength, are expected to become important. Since we are studying a low energy effective action this limit thus seems irrelevant at first sight. Here we take the attitude, however, that there may be features of the high energy theory that are of topologic and algebraic nature and may be captured in such a limit. Alternatively, since we do not prove this, one may also study the limit of the model considered in its own right (i.e. independent of its relation to string theory).

We again consider the type-IIB theory, which means that p is restricted to be odd. We are interested in the first few terms in an ε-expansion of the action. To that end we rewrite the Born-Infeld part of (2.5) as

$$S = g_s^{-1} \int d\xi^1 \wedge \cdots \wedge d\xi^{p+1} e^{-\Phi} \Big[\det \gamma \det \left\{ (\varepsilon B + F)_\alpha{}^\beta \right\} \det \left\{ (\delta + \varepsilon (\varepsilon B + F)^{-1})_\beta{}^\gamma \right\} \Big]^{1/2} . \tag{4.1}$$

Making use of the expansion[6]

$$d\xi^1 \wedge \cdots \wedge d\xi^{p+1} \sqrt{\det\{(\varepsilon B + F)\}} = \frac{1}{l!} (\varepsilon B + F) \wedge \cdots \wedge (\varepsilon B + F) \,, \tag{4.2}$$

where $2l = p + 1$, we expand the full action (2.5) as follows

$$\begin{aligned} S = \frac{g_s^{-1}}{l!} \int \Big\{ & (e^{-\Phi} + g_s C^{(0)}) F \wedge \cdots \wedge F + \\ & + l\varepsilon ((e^{-\Phi} + g_s C^{(0)}) B + g_s C^{(2)}) F \wedge \cdots \wedge F + O(\varepsilon^3) \Big\} . \end{aligned} \tag{4.3}$$

We see that the leading term corresponds to a coupling to a D-instanton. With $q(X) \equiv e^{-\phi} + g_s C^{(0)}$ it has the form

$$\int q F \wedge \cdots \wedge F \,, \tag{4.4}$$

which displays its topological character: when $q =$ const. it is proportional to the Chern class of the U(1) field, and it is always independent of a metric.

The subleading is a (generalized) Chern-Simons type term. Further we note that the form of the action (4.3) preserves $\mathrm{SL}(2,\mathbb{Z})$-invariance. In fact, had we only considered the Born-Infeld action in this limit and then imposed $\mathrm{SL}(2,\mathbb{Z})$-invariance, we would have discovered the $C^{(0)}$ and $C^{(2)}$ couplings.

5. Discussion

We have studied two limits the Born-Infeld action for D-branes in a given fixed background. As a basis for this we used an action with rescaled fields and tension (ε). The strong coupling limit we took to be defined by $\varepsilon^{\frac{p+1}{2}}/g_s \ll 1$ with ε fixed and the high energy limit we studied was $\varepsilon \to 0$.

[6] This corresponds to the expansion of the determinant of a $2l \times 2l$ skew matrix as the square of the pfaffian.

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In the first limit we were able to solve the V^i, W^i, A_i and X^μ equations for $C^{(n)} = 0$, $n = 0, \ldots, p$ and recover the picture of the world volume as foliated by strings from [1]. Restrictions on the geometry led to the possible constituent strings going off to infinity (D1-branes) or being closed.The general case proved to be considerably more complicated due to the presence of the current built from the $C^{(n)}$'s, although we saw that the "string parton" picture survived in a particular example. In that example we found the additional option of the constituent strings being open but ending on the D-brane sources of the $C^{(n)}$ background fields.

In the second limit, we found an ε-expansion whose leading term is topological. We also noted that SL$(2, \mathbb{Z})$-invariance predicts the form of the $C^{(0)}$ and $C^{(2)}$ couplings in the next to leading term. We also noted that interpreting this limit as taking the radius $R \to 0$ in the AdS picture, this limit should probe physics away from the large N limit.

To proceed further with both these expansions one would need to specify the background. In fact, solving the A_i and X^μ equations in the second limit order by order in ε requires knowledge of the α'-dependence of the background. This is a possible direction for future research. It would also be interesing to compare our results to other ways of extracting string dynamics from Born Infeld theory, e.g. the Born-Infeld strings of [9]

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References

[1] U. Lindström and R. von Unge, *A picture of D-branes at strong coupling*, *Phys. Lett.* **B 403** (1997) 233 [hep-th/9704051].

[2] H. Gustafsson and U. Lindström, *A picture of D-branes at strong coupling. II: spinning partons*, *Phys. Lett.* **B 440** (1998) 43 [hep-th/9807064].

[3] A.A. Zheltukhin, *Tension as a perturbative parameter in non-linear string equations in curved space-time*, *Class. and Quant. Grav.* **13** (1996) 2357 [hep-th/9606013]; S.N. Roshchupkin and A.A. Zheltukhin, *Variational principle and a perturbative solution of non- linear string equations in curved space*, *Nucl. Phys.* **B 543** (1999) 365 [hep-th/9806054].

[4] E. Bergshoeff and P.K. Townsend, *Super D-branes revisited*, *Nucl. Phys.* **B 531** (1998) 226 [hep-th/9804011].

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[5] C.M. Hull, *String dynamics at strong coupling*, *Nucl. Phys.* **B 468** (1996) 113 [hep-th/9512181].

[6] W. Taylor IV, *Lectures on D-branes, gauge theory and M(atrices)*, Trieste summer school 1997, hep-th/9801182.

[7] M.R. Douglas, *Branes within branes*, hep-th/9512077.

[8] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[9] J.C. Callan and J.M. Maldacena, *Brane dynamics from the Born-Infeld action*, *Nucl. Phys.* **B 513** (1998) 198 [hep-th/9708147].

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